AN OPTIMIZATION APPROACH FOR 2D FINITE ELEMENT MESH SMOOTHING

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Finite element analysis (FEA) is useful in many application areas. However, if the analysis is performed on a low-quality mesh, the FEA can be inaccurate. Various methods, including mesh smoothing approaches, to improve mesh quality have been studied. One well known mesh smoothing approach is the Laplacian smoothing, which can generate slivers or invalid elements in some cases. One improved mesh smoothing approach is Zhou and Shimada's algorithm [1], which uses a torsion spring system scheme to move mesh nodes. In particular, it optimizes the interior angles of all mesh polygons that have vertices linked to an internal node of the mesh. These angles are optimized by moving the linked internal node, however the new position of the internal node is chosen by a non-optimized scheme.

In this paper, we present a new optimization algorithm that will better optimize the new locations of all nodes. The first step of our algorithm is to optimize the energy E of a torsion spring system that can be expressed for a polygon P as: E= $\sum_{i=1}^{2n} \frac{1}{2}K(\theta_i)^2$, where K is a constant, n is the number of vertices in P and θ_i is the angle between polygon edge E_i and the segment linking the internal node N with vertex V_i. (Fig. 1 illustrates a polygon P defined by vertices linked to an internal node N.) The second step of our algorithm is to optimize with the Gauss-Newton algorithm the objective function S: $S = \sum_{i=1}^{n}[distance(\hat{N}, L_i)]^2$, where L_i is the bisectional line of the internal angle of Polygon P at Vertex V_i and $\hat{N}$ is the new position of the internal node. In application to real meshes, our new smoothing algorithm has been found to give better results than both the Laplacian smoothing and Zhou and Shimada's smoothing, especially in terms of avoiding invalid and sliver-like elements. Results for one original mesh (i.e., the meshes generated by the Laplacian smoothing, Zhou and Shimada's smoothing and our new smoothing algorithm) are shown in Fig. 2.

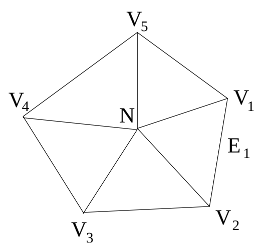

Figure 1: Polygon P

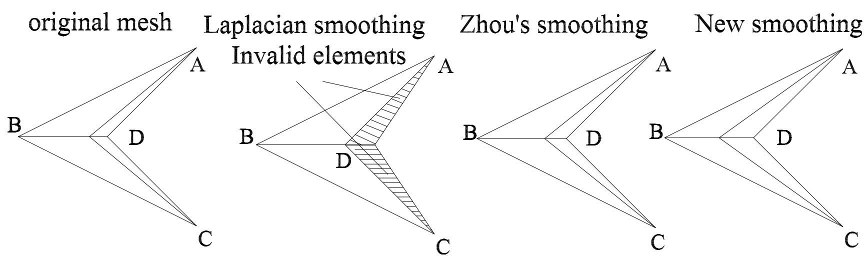

Figure 2: Comparison of smoothing algorithm for one mesh

References

[1] T. Zhou, K. Shimada, "An Angle-Based Approach to Two-Dimensional Mesh Smoothing," *Proceedings, 9th International Meshing Roundtable*, p. 373-384, 2000.